Calculus I

Challenge Homework Set I

May 7, 2025

Provide **handwritten** answers on a separate sheet of paper. Typed answers will not be accepted. For full credit correct answers should be clear, legible, include explanations for your reasoning, and show all relevant work. You are allowed to make use of outside resources, including the internet, and friends, but you must cite your sources. **Textbook Problems**:

Ch 4: 113-128, 316, 322, 338-340

i) Find the critical points of the following functions, and evaluate whether the critical points are maxima or minima.

   a) $\cos(2x)$

   b) $\sin(5x)$

   c) $\sin(|x|)$

   d) $|\cos x|$

ii) In this problem we consider optimizing the volume or surface area of certain shapes. **Hint:** Draw pictures!

   a) Find the largest volume of a cylinder that fits into a cone of radius r and height h.

   b) Find the dimensions of a cylinder with volume $16\pi\,\mathrm{m}^2$ that has the largest surface area.

   c) Find the dimensions of a right cone with surface area $4\pi\,\mathrm{m}^2$ that has the largest volume.

   d) Suppose that total surface area of a cube and and sphere is $1\,\mathrm{m}^3$. Find the dimensions of the cube and sphere such that the total volume is maximized.

iii) For this problem recall that (x_1, y_1) and (x_2, y_2) are two points in the plane, then the distance between them is given by:

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

   Using this, answer the following questions:

- Where is the line $y = 5 - 2x$ closest to the origin?
- Where is the parabola $y = x^2$ closest to the point $(2, 0)$?
- Where is the cubic $y = x^3$ closest to the point $(2, 2)$?

iv) An object with mass m is dragged along a horizontal plane by a force acting along a rope attached to the object. If the rope makes an angle θ with a plane, then the magnitude of the force is:

$$F = \frac{\mu m g}{\mu \sin\theta + \cos\theta}$$

where g is the acceleration due to gravity, and μ is a dimensionless constant called the coefficient of friction. For what value of θ is F minimized?