Calculus I

Challenge Homework Set III

April 22nd, 2025

Provide **handwritten** answers on a separate sheet of paper. Typed answers will not be accepted. For full credit correct answers should be clear, legible, include explanations for your reasoning, and show all relevant work. You are allowed to make use of outside resources, including the internet, and friends, but you must cite your sources. **Textbook Problems**:

Ch 3: 311,313,315,325, 326, 327, 347, 349, 351, 353

Challenge Problems:

i) A ladder 10 feet long is leaning against a vertical wall. The bottom of the ladder is sliding away from the wall at a rate of 2 ft/s. How fast is the top of the ladder sliding down the wall at the moment when the bottom is 6 feet from the wall? Hint: Use the Pythagorean theorem, and differentiation!

We have that the y position of the top of ladder, adn the x position of the bottom ladder must satisfy the Pythagorean theorem:

$$x^2 + y^2 = 100$$

Since the ladder is sliding down the wall, we have that x and y are dependent on time, and are thus functions of time. If we take a time derivative of x^2, or y^2 we must then apply the chain rule:

$$\frac{d}{dt}(x^2) = 2x \cdot \frac{dx}{dt} \qquad \text{and} \qquad \frac{d}{dt}y^2 = 2y\frac{dy}{dt}$$

Taking a time derivative of both sides of the relation $x^2 + y^2 = 100$, we thus get:

$$2x \cdot \frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

Now, we know that the bottom part of the ladder is sliding away from the wall at a constant rate of 2 ft/s hence $dx/dt = 2$. Moreover, when $x = 6$, we must have that:

$$36 + y^2 = 100 \Rightarrow y^2 = 64 \Rightarrow y = 8$$

We thus see that:

$$2 \cdot 6 \cdot 2 + 2 \cdot 8 \cdot \frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = -\frac{3}{2}\,\text{ft}\,/\,\text{s}$$